Diffusion of information, choices, norms etc.

A large number of problem in social sciences and economics can be approached from the same perspective and methodogy. They basically concern the process of diffusion of information, or most often beliefs, across a population of individuals. On can be interested into such processes for their own interest, or otherwise, as in economics of choices, consider that in situations of uncomplete information, agents choices are influenced by the choices, or beliefs, of other agents whom they trust.

Among such problems are:

- Information or belief contagion;
- epidemics of infections, biological or computer viruses;
- social choices, under bounded rationality which might result from neighbours opinions or choices; these choices might concern products, strategies (e.g. investments), adoption of new rules or technologies (e.g. the green revolution);
- “positive externalities” in the choice of standards, software etc.
- political choices.

The problem might be considered from the perspective of buyers (resp. adopters, investors, voters..) or sellers (resp. sales representatives, agencies, political parties) or from both.

A typical instance of a model in this class implies to define the topology of the social substrate:

- what we call full mixing, any agent being possibly connected to any other agent.
- a network structure, often represented by a lattice (large clustering) in simulations, or any other kind of “more realistic” network such as small worlds or scale invariant nets.

In fact, the observed dynamics also depend upon the nature of what is transmitted:

- binary options;
- continuous opinions;
- multi-dimensionnal traits (e.g. Axelrod model of the diffusion of culture).

1 Binary options

1.1 Verlhurst equation

One of the earliest model of epidemics of pathogens, culture, sales etc. is Verlhurst equation, often called the logistic equation, proposed to describe population growth in 1846. The logistic equation is widely used in population dynamics and its time discrete version was used to demonstrate chaos by Feigenbaum (http://www.ukmail.org/˜oswin/logistic.html).

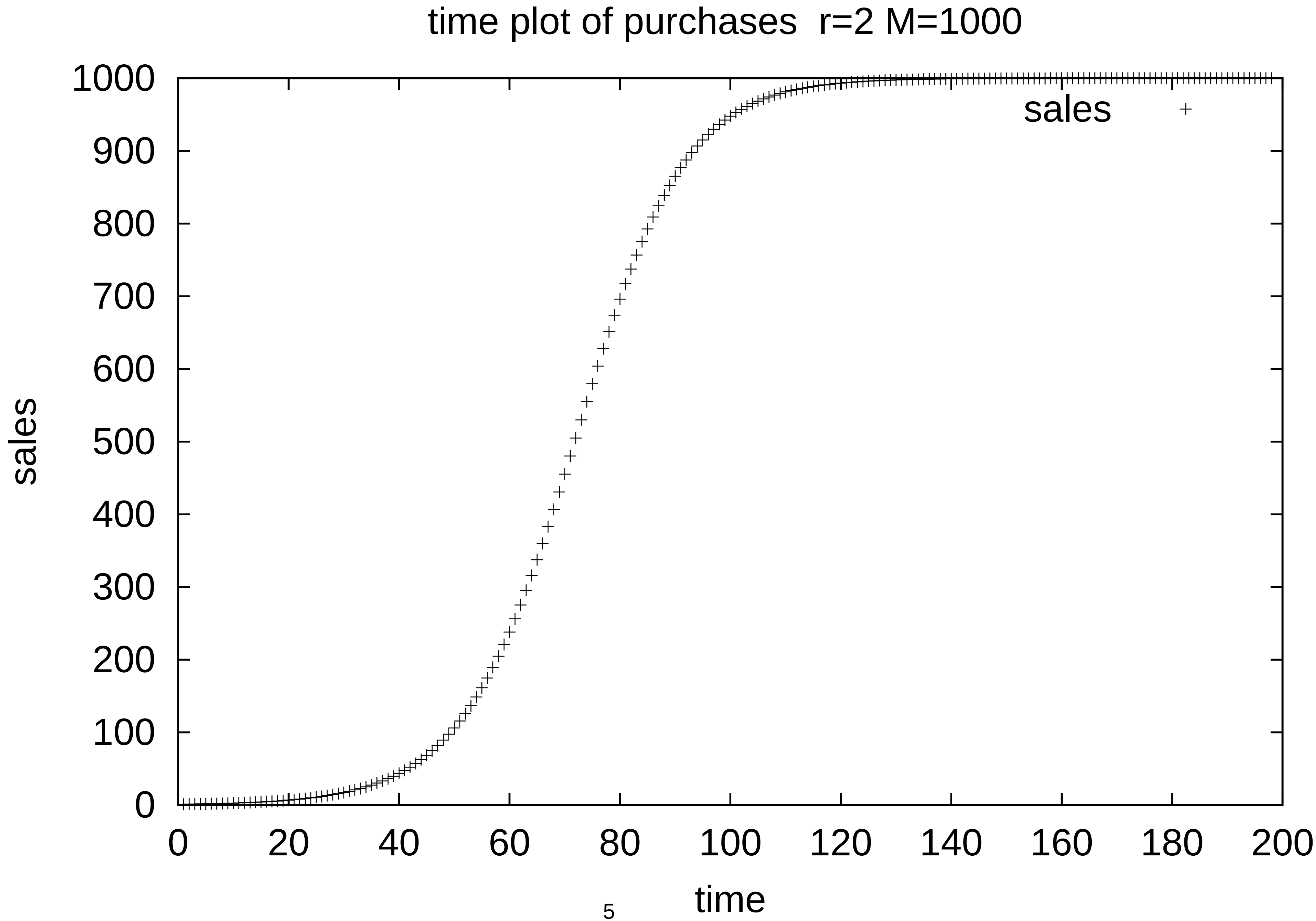
The continuous time version in epidemics is easy to interprete :

$$dx/dt = rx(1 - X/M)$$

the infected population increase is proportional to the infected population x, through an infection rate r, but is limited to the total population M.

The logistic equation is the global description of a set of binary processes describing the infection process as resulting from encounters between infected sujects (density x) and susceptible subjects (density M-x).

The integration of this simple differential equation gives well known sigma curve dynamics of infection. The infection starts slowly, increases exponentially, but eventually saturates when all the population is infected. This elementary dynamics can be (and was) applied to pathogens, ideas, markets, adoption of new practices etc. .


time plot of purchases r=2 M=1000
sales
time
0
100
200
300
400
500
600
700
800
900
1000
0
20
40
60
80
100
120
140
160
180
200

The sigma time plot or its derivative is widely used to describe the evolution of sales of a new products.

The logistic equation with the addition of an external harvest often describes the dynamics of the resource in bio-economics of renewable resources such as fisheries.

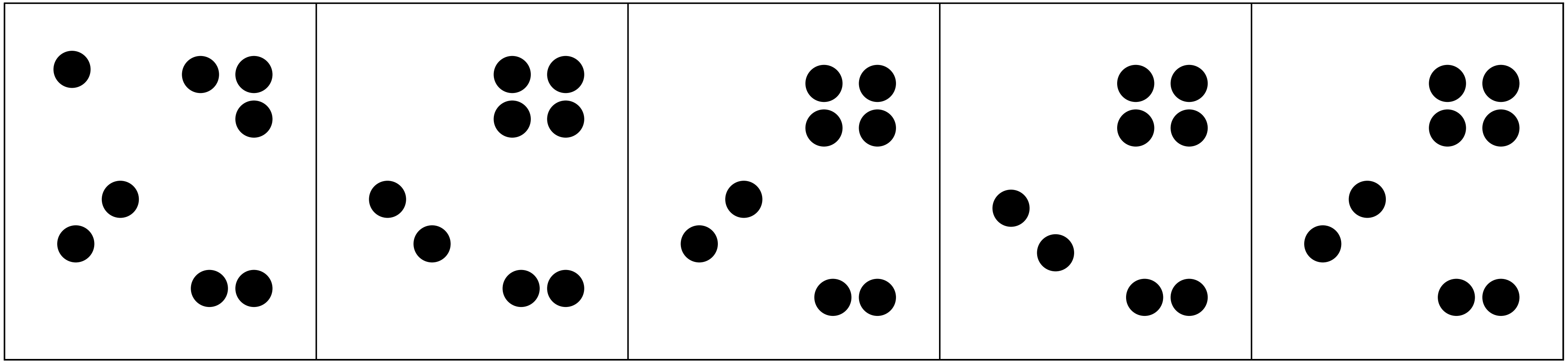
1.2 Lattice growth

If we suppose that we can represent the underlying social structure by a regular lattice, the dynamics is isomorph to crystal growth dynamics studied by physicists. The lattice structure often disputed by economists and others as non realistic, still offers one property missed by random graphs, the clustering property, represented by short loops or as the popular sentence says: “the friends of my friends are my friends”. It is often used because it naturally offers simple visualisation. Visualising dynamics on more complex networks is very hard since causal links are missing.

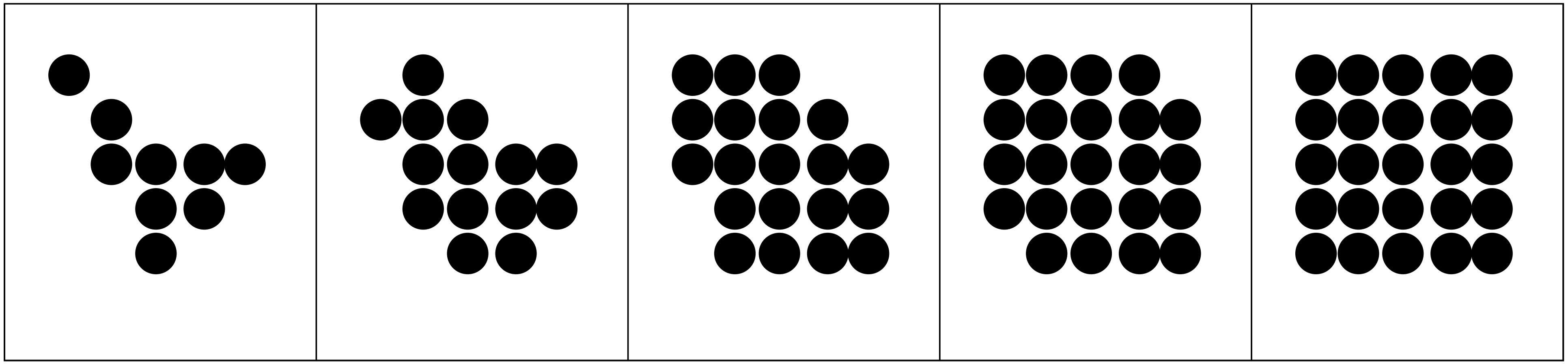
The standard model on lattice growth is based on counter automata.

Let’s for instance consider a square lattice, with a von Neumann neighborhood: each site is a boolean automaton with five input, from his four neighbours N, S, E and W, + itself. The state of the site, 0 or 1, is obtained by comparing the sum of its neighbours’ states (hence the name counter) to a threshold. If the sum is larger than the threshold, the state of the automaton is set to 1 and to 0 otherwise. Depending from the threshold several dyanmics are observed. Negative or low thresholds favour the growth of 1 regions, threshold above 4 favouring 0 regions.

- If the threshold is negative, or 0, growth proceed in one single time step.
- If the threshold is 1, growth proceeds slowly from isolated (or not) seeds.
- The two series of configurations below show the growth of 1's patterns in a sea of 0's for a threshold of 2: two neighbours at state 1 are indispensible to take state 1.



Isolated configurations



Filling the convex hull

Growth is limited to the convex hull of the initial configuration.

Starting form random initial configuration of 0's and 1's, invasion of the whole lattice may proceed for a sufficiently large density of initial 1's. But locally, certain seed configuration should be present: the early birds or hopeful monsters,

For thresholds 3 and above, the situation is inverted between 0's and 1's: 0's region may grow from uniform one regions if seed configurations are present.

Let us note some differences between the full mixing and the lattice case:

- Initially, exponential growth is ensured in the full mixing case as soon as at least one adopter is present; in the lattice case, growth needs seed configurations, and proceeds more slowly (linearly in two dimensions). But empirical checks of the scaling would be hard to implement because of the few initial purchases!
- In both cases saturation is observed, but there is a threshold in initial densities for the lattice case to observe full invasion.

The discussion for the von Neumann neighborhood can be generalised to any neighborhood.