Preprint of a Chapter to appear in the book:

Cognitive Economics

P. Bourgine and J.-P. Nadal Editors,

Springer-Verlag

Springer

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Contents

1 From Agent-Based Computational Economics towards Cognitive Economics

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Abstract. This Chapter provides a short introduction to Agent-based Computational Economics (ACE), in order to underline the interest of such an approach in cognitive economics. Section 2 provides a brief bird's eye view of ACE. In section 3, some interesting features of the Santa-Fe Approach to complexity are then introduced by taking simple examples using the Moduleco computational laboratory. Section 4 underlines the interest of ACE for modelling and exploring dynamic features of markets viewed as cognitive and complex social interactive systems. Simple examples of simulations based on two cognitive economics models are briefly discussed. The first one, deals with the so-called exploration-exploitation compromise, while the second deal with social influence and dynamics over social networks .

1.1 Intoduction

Leigh Tesfatsion [67] defines Agent-based Computational Economics (ACE) as "*the computational study of economies modelled as evolving systems of autonomous interacting agents. Starting from initial conditions, specified by the modeller, the computational economy evolves over time as its constituent agents repeatedly interact with each other and learn from these interactions*". A growing proportion of ACE uses "*computational laboratories*" (CL); i.e a *multi-agent framework*, based on object-oriented languages. In such a framework, the modeller has few codes to write and can use different kinds of preexistent agent types, interactions - communications structures, rules etc. CL allows us to study complex adaptive systems with multiple interacting agents by means of *controlled and replicable experiments*, usefull to compare different models in the same framework. Moreover, CL provides "*a clear and easily manipulated graphical user interface that can permit researchers to engage in serious computational research even if they have only modest programming skills*" [66]. In the *Moduleco* CL [55] used for this chapter, ACE embodies the two sub-perspectives of cognitive economics: the "*eductive* ", one and the "*evolutionist*" one (see Walliser, this book). More specifically, in this chapter, the "evolutionist" perspective is taking closer to the Santa Fe' approach (SFA), which is related to the "complex adaptive systems" paradigm [3,6]. Therefore, according to a wide usage in the literature, we refer to this later approach as being an *evolutionary* one, but talking of *evolutionist* to refer specifically to the corresponding methodological perspective of cognitive economics proposed by Walliser, in this book. A key feature of those models is viewing the emerging order as a product of the system dynamics (system attractor), and more specifically of its element interactions [11]. At this time, the *eductive* perspective is less developed within ACE, but some authors are attempting to develop some tasks on the evolution of learning representation, but mainly in an evolutionist perspective.

This chapter provides a brief bird's eye view of ACE, and complexity-related concepts (section 2 and 3). Section 4 deals with dynamics over social networks. The effect of communication structures' topologies upon dynamics is discussed using very simple examples.

1.2 Agent-based Computational Economics and multi-agent systems in economics

This section provides a bird's eye view of the principles and applications of ACE in economics, and underlines the interest of ACE for modelling markets viewed as cognitive and complex social interactive systems.

1.2.1 Agent Based Computational Economics

Because many surveys about ACE are available [63–66] we only outline in this section the main topics of this research area, some references and questions raised by this growing literature. Three special journal issues in 2001 devoted to ACE provide a large sample of current ACE research ([1,2], IEEETEC 2001). Tesfatsion roughly divides this research area into eight topics : (i) Learning and the embodied mind; (ii) evolution of behavioural norms; (iii) bottom-up modelling of market processes; (iv) formation of economic networks (v) modelling of organisations, (vi) design of computational agents for automated markets; (vi) parallel experiments with real and computational agents (viii) building ACE computational laboratories [66]. In addition, LeBaron [38] proposes some suggested readings in agent-based computational finance and a "builder's guide" for such models [39]. Finally, Axtell and Epstein, authors of a book which has become a reference in this field : *Growing Artificial Society, Social Sciences from the Bottom Up* [21], provide methodological issues ([8,9] see also, among others, [23]. Let us note that topic (i) is close to the *eductive* sub-perspective of cognitive economics, while topics (ii) and (iv) are more related to the *evolutionist*' sub-perspective. Topic (iii) is concerned as much with *eductive* as with *evolutionist*, because the market process involves both individual and collective learning .

Why Agents? For Axtell [8] there are three distinct uses of ACE: (1) classical simulations, (2) as complementary to mathematical theorising and (3) as a substitute for mathematical theorising. In the first case, ACE is used on the one hand as a friendly and powerful tool for presenting processes or results, or, on the other, as a kind of Monte-Carlo simulator, in order to provide numerical results. The latter case is often used by the evolutionary approach (like Dosi, Marengo, Yildizoglu, among others...) in the case of intractable models, specially designed for computational simulations . In this chapter, we focus on the middle case, when ACE is used as a *complement to mathematical theorising.* Axtell mention several cases relevant for this category. This is, for example, the case when an equilibrium exists but is uncomputable or is not attained by bounded rational agents, or is unstable, or realised only asymptotically in the (very) long run. This is also the case where some parameters are unknown, making the model incompletely solvable.

Cognitive economics is specially concerned with the last topic, where the equilibrium position is known only for a simple interaction network. It is the case, for instance with statistical mechanics - related models reviewed by Phan *et al.*(this book), such as, for example, [51,56]. In this latter, we know analytically the optimal asymptotic monopolist pricing in two polar cases: without externality or with global externality. Analytical results may be possible for the homogeneous regular case. But in the mixed case (including the so-called "small world", to be presented in the following) characterised by both highly local and regular connections and some long range, disordered connections, numerical (statistical) results are often the only possible way.

From an *eductive* point of view, the highly path dependent process of diffusion upon such networks involves learning i.e. (i) belief revision (for instance, in the case when a monopoly faces customers randomly distributed on a given network, even if the initial distribution is well known, as in [51,56] see also, [40,41] or (ii) eductive co-ordination in the case of rational agents playing a game with their nearest neighbourhood (as in the Blume-Brock-Durlauf approach reviewed by Phan *et al.*, this book).

From an *evolutionary* point of view, attention may focus upon "classical" complex adaptive systems dynamics [73] with a SFA flavour. The following two sub-sections introduce some of these concepts, such as emergence, attractor, phase transition and criticality based on examples taken from Moduleco.

1.2.2 Simulating implies understanding [20]: markets viewed as cognitive and complex social interactive systems modelled by the way of ACE on multi-agent software.

Cognitive economics is an attempt to take into account the incompleteness of information in the individual decision making process, on the one hand, and the circulation and appropriation of information within social networks , on the other hand. Because of incompleteness of information, in cognitive economics, learning is a central feature both at individual and collective levels. Multi-agent modelling and simulation of complex adaptive systems are complementary tools as well as experimental economics to investigate this field.

Following Kirman (in this book) a market can be viewed as a *complex and cognitive informational and interactive system*, socially produced. From this perspective, ACE is a promising approach for investigating market mechanisms [69,35]. More specifically, multi-agent framework appears to be a pertinent tool for understanding observable market phenomena. In such a system, buyers as well as sellers may be represented by a suitable software agent. Each agent is then linked by communications structures to other entities of the systems. In this way, such an agent may exchange information with his environment, to adapt his behaviour given this information (individual learning). As a consequence, each agent contributes in this way to the adaptation of the whole system (collective learning, following [19,70].

To explore market properties in this approach, knowledge of the general properties of the complex system dynamics [73] is the first step. At a lower level of abstraction, a cognitive economics approach gives more consistency to both individual behaviour and social representations (Orléans in this book), taking the more generic properties as given. An inter-disciplinary multi-level interpretation of both properties and assumptions requires specific reflection, like, for instance, Phan *et al.*, this book, for a discussion of significance in the use of statistical physics by economists.

The general conceptual framework for such research was mainly constituted during the 90's, even if some important contributions were produced in the two decades earlier. Multi-agent systems [22], which are well adapted to this approach, were originally strongly linked with "artificial life" [37,36,14]. Multi-agent platforms are oriented towards simulations and *in silico* experimentations. The most famous multi-agent platform is SWARM, initiated by Langton (see [46] for applications to the economic field). Others multi-agent platform dedicated to economics problems are among others, ASCAPE [54], CORMAS and LSD [68]. For this chapter, we use MODULECO, a multi-agent plateform, built in java, an object programming language.

In ACE, economic agents are generally heterogeneous in some attribute. When agents have some heterogeneity by themselves, without any interaction, we call this characteristic *idiosyncratic heterogeneity.* When agents interact, the combination of their adaptive or learning capacities together with their insertion within a specific structure of interaction generally drive the agents towards heterogeneous individual trajectories, even if they are initially homogeneous. We call this situation *interactive heterogeneity.* Beyond the analytical results that it is sometimes possible to obtain in generally very simplistic cases, it is interesting to undertake *in silico* experimentations. This means simulations of more complex cases for which analytical results do not exist. For example, [56] explore a large range of network structures for a discrete choice model in a monopoly case with (and without) externality. Simulation allow exploration between two polar cases, for which we have analytical results; that is, the case without externality and the case with global externality (see section 4 and Phan *et al.*, this book). These kinds of models grant a significant place to the circulation of information and the adaptive phenomenon. As a consequence, the study of processes matters as much as the analysis of the asymptotic states to which processes may eventually lead.

Following the method suggested by the autonomous agent systems literature, ACE first produces *generic results*, i.e. common to natural, living or human systems. Secondly, these results, which are highly abstract, must be reinterpreted in the field of a specific scientific domain, by a specific discussion of all assumptions, postulated relationships and behaviours. Some additional assumptions may be added or some others removed. The ultimate step is the most difficult to formalise. Human agents have a very specific characteristic, which radically distinguishes then from particles or ants. A human agent is an eductive one. A human agent may integrate emerging phenomenon in his representations and change his behaviour according to this revising process. So, a first step in modelling social phenomena by a large multi-agent framework, is to ask (following [17]: when is individual behaviour (like conceptual capacities) negligible or when is it decisive?

1.3 Basic concepts of multi-agent systems with network interactions.

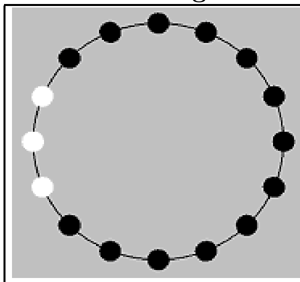
Complex adaptive systems dynamics [73,61] may change with circumstances. There is no proportionality between cause and effect. A very interesting feature of such a system is classical in the physics of disordered systems: phase transition ([18], Galam, this book, [26] for an economist's point of view). In the simplest case of phase transition, the system only bifurcates between two opposite states, but many other dynamic behaviours may arise. Physicists attribute sometimes such phenomena to symmetry breaking ([4], Galam in this book). Broken symmetry gives rise to the appearance of a new phenomenon that did not exist in the symmetric phase. Complex adaptive systems, strongly non-linear, in many cases resist classical methods of analysis (reductionism) and yet they may be governed by very simple rules.

In this section, we outline the main features of SFA by taking some simple examples using the Computational Laboratory "Moduleco", we introduce as simply as possible three basic concepts of complexity in multi-agent systems. First opening by phase transition and complex dynamics in the case of a simple spatial evolutionary game, we introduce next the role of the topology of communication structures in collective dynamics, with the so-called "small world", within the same evolutionary game framework. We raise finally the question of emergence with the Schelling's Model of Segregation [58–60].

1.3.1 Basic concepts of multi-agent systems (1): complex dynamics in adaptive systems

When individual actions are made to be interdependent, complex dynamics may arise. That is the case, for instance, when agents locally interact over a specific network. Kirman, this book, discusses this question for market studies. In order to illustrate such a phenomenon, a very simple model of the spatial prisoner dilemma is presented here. The simplest version (on a one dimensional periodic lattice) exhibits only a phase transition between two symmetric states: complete defection and complete co-operation. More complex behaviour may arise when the connectivity increases, like in the [47,48] model, where agents interact on a two dimensional periodic lattice (torus), or when the network is not a regular one, as in section 4. The introduction of random noise may also produce different results, but here we only consider the determinist mechanism. In the generic model, agents play a symmetric

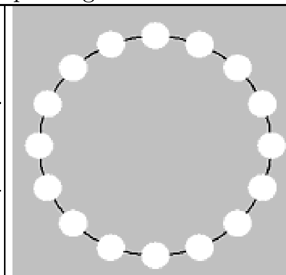
Fig. 1.1. the simplest one dimensional spatial game



$176 > X \geq 92$: defection is contained in a "frozen zone"

$(J1, J2)$	$J1/S1$	$J1/S2$
$J2/S1$	(X, X)	$(176, 0)$
$J2/S2$	$(0, 176)$	$(6, 6)$

S1 : co-operation (black)
S2 : defection (white)



$6 < X \leq 91$: : the whole population turns to defection

game (here, a prisoner dilemma) with each of their "neighbours" on a lattice. At a given period of time, each agent plays the same strategy (S1 : co-operation or S2 : defection) in all these bilateral games. At the end of the period, each agent observes the strategy of his neighbours and the cumulated payoff of their strategy. But the agent has no information at all about the other games played by his neighbours. He observes only the cumulated payoff linked with this strategy.

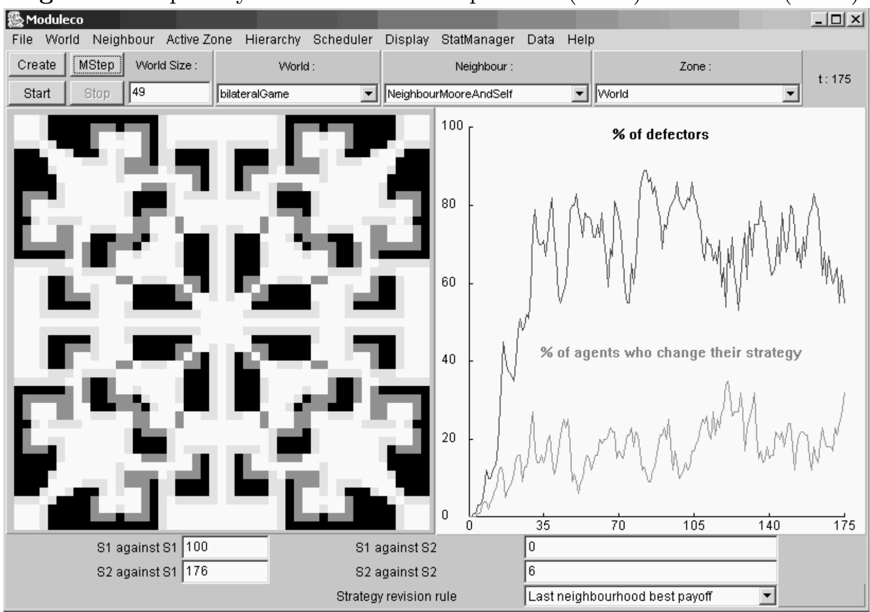
At each period of time, agents update their strategy, given the payoff of their neighbours. Assuming myopic behaviour, the simplest rule is to adopt the strategy of the last neighbourhood best (cumulated) payoff. Another rule (used by [28] is to adopt the strategy of the last neighbourhood best average (cumulated) payoff. This latter rule is less mimetic, because one may interpret this revision rule as a kind of estimator of the expected cumulated payoff of a given strategy (for the model maker, that is a conditional expected payoff given the strategies of the neighbour's neighbourhood). Finally, bilateral games plus the revision rule constitute a special kind of evolutionary game [49].

In the simple model of Figure 1.1, agents play a symmetric game (prisoner dilemma) with each of their two neighbours on a circle (one-dimensional, periodic lattice). The revision rule is the last neighbourhood best cumulated payoff. If the payoff of the co-operation against themselves is sufficiently high

(S_1 against $S_1 > 91$), defection (S_2) is contained in a "frozen zone" of 3 agents. In other cases (S_1 against $S_1 < 92$), the whole population turns to defection. For $N \geq 3^2$ this result is independent of the number of agents

In [47,48], there is a population of co-operators on a torus (two dimensional, periodic - in our example : $49^2 - 1 = 2400$ co-operators). Each agent plays with his eight closest neighbours (Moore neighbourhood). The revision rule algorithm takes into account the payoff of the player's strategy against himself. As in the previous example, one makes an agent become temporarily

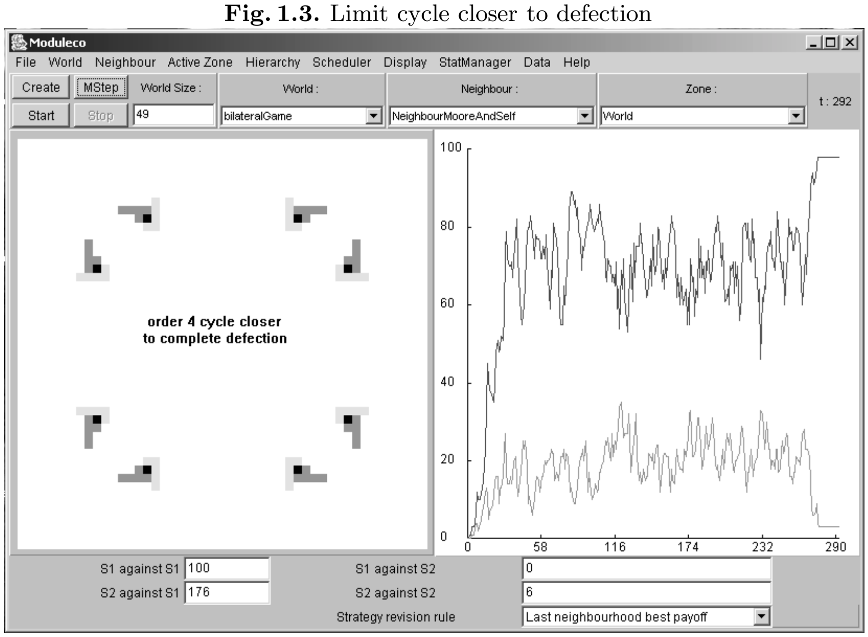
Fig. 1.2. Complex dynamics between co-operation (black) & defection (white)



Light grey : defector who turns to co-operation ($S_2 > S_1$)
Dark grey : co-operator who turns to defection ($S_1 > S_2$)

a defector. For a sufficiently high payoff of the co-operation against himself (S_1 against $S1 \geq 101$) the defection (S_2) is to be contained in central zone of 9 agents. For $113 \geq S1 \geq 101$, it is a "frozen zone" of defectors, for $129 \geq S_1 \geq 114$ a cycle of period 3 and for $157 \geq S_1 \geq 130$, a cycle of period 2. This result holds for all populations, from 6^2 agents. At the contrary, for a weak payoff of co-operation against itself the whole population turns to defection after short transitory dynamics. For instance, for (S_1 against $S_1 = 94$) total defection arises after 30 periods. For an intermediary payoff (in this case 99-100), the dynamic trajectory becomes quasi-chaotic and produces beautiful geometrical figures (Figure 1.2). In this particular case (S_1 against $S_1 = 100$), the trajectory converges (Figure 1.3) towards a cycle of period 4 after 277 iterations. Such a phenomenon arises for a sufficiently large population. For instance, for this set of payoffs at least 432 agents are needed in order to induce a cycle of order 2, after 2138 iterations of chaotic behaviour. In the special case of this model by May, Nowak, results do not really make

Fig. 1.3. Limit cycle closer to defection



sense in economics. Nevertheless, three important phenomena appear in this simple case. First, if agents' behaviour are interrelated, strictly deterministic and identical agents with very simple individual behaviour may produce both heterogeneity at the micro level and complex dynamics at the macro level. Next, some critical values around the symmetric point (between the co-operative "phase" – or order – and the defective one) play an important role in such dynamics. Finally, the nature of the dynamics depends on the topology of interrelations.

1.3.2 Basic concepts of multi-agent systems (2): the role of the topology of communication structures in collective learning: the so-called "small world"

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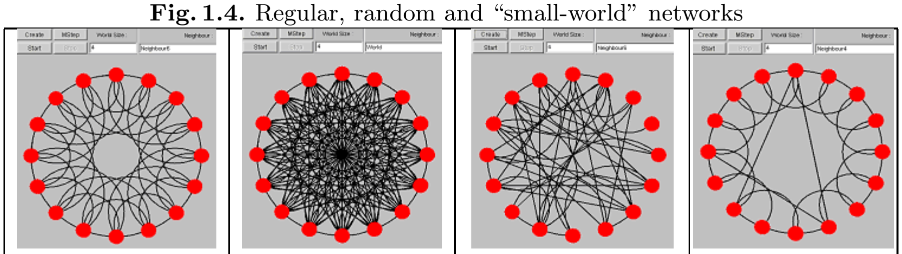
Following an important body of literature in the field of socio-psychology and sociometrics, initiated by Milgram [50], the "six degrees of separation" paradigm of a "small-world" , Watts and Strogatz [72] proposed a formalisation in the field of disordered systems. The original Watts and Strogatz (WS) "small world" starts from a regular network where n agents are on a circle (dimension one, periodic lattice) and each agent is linked with his $2.k$ closest neighbours. In the WS rewiring algorithm, links can be broken and randomly rewired with a probability p. In this way, the mean connectivity remains constant, but the dispersion of the existing connectivity increases. For $p = 0$ we have a regular network and for $p = 1$ a random network.

Fig. 1.4. Regular, random and "small-world" networks



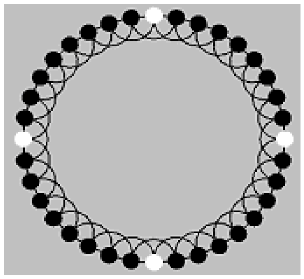
(a) Regular network connectivity $k = 6$	(b) Regular network full connectivity	(c) Random network connectivity constrained	(d) Small-world $k = 4$; 3 links rewired

Intermediate values between 0 and 1 correspond to the mixed case, where a lower p corresponds to a more local neighbour-dependent network. In Moduleco, the actual algorithm took h nodes, broke i links for each of these nodes and randomly rewired the broken links with other nodes. We have a parameter $q = (h_i)/n$ which plays a similar role to p (Figure 1.4). A large scope of small world properties is now well known [71,52]. Zimmerman, in this book, provides a short review of basic features. [10] provide a typology of small world, with related properties, including both WS and some varieties of "scale free" topologies.

In economics, the small world has been applied to bilateral games [28,27], the knowledge and innovation diffusion processes (see Zimmermann, this book) and market organisation [74,56,51] The following example is drawn from work in progress [53] to illustrate the power of rewiring in changing the interactive environment. For the spatial prisoner dilemma game (and a larger class of bilateral games), Jonard *et al.* [28] have established (for the best average payoff rule) that the stability of co-operative coalitions depends on the degree of regularity in the structure of the network. In the following example, co-operation is unsustainable within a regular network, but become sustainable within a rewiring disorder.

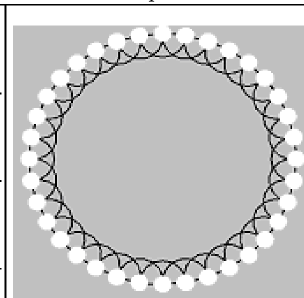
The core of the model is the same as that of the spatial prisoner dilemma, but with a one dimensional - periodic neighbour 4 structure (on a circle). To be clear, we have limited the population to $N = 36$ agents (32 co-operators for 4 defectors). According to the best neighbourhood payoff rule, each agent chooses the best cumulated payoff strategy in the neighbourhood. The aim of this exercise is to improve the strength of a network against accidental defection. That is, four temporary defectors are symmetrically introduced into the network. When the network is regular, defection is the winner strategy, and diffuses to the whole population (Figure 1.5) In some cases, changes in the structure of the networks by minor modifications in the neighbourhood of some agents allow co-operation to protect against defection. The number

Fig. 1.5. Symmetric introduction of defection in a network of co-operators



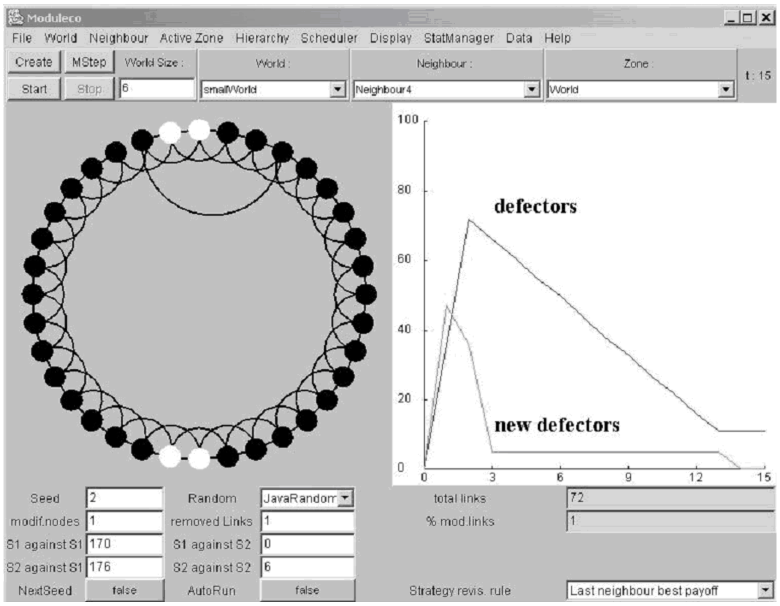
$(J1, J2)$	$J1/S1$	$J1/S2$
$J2/S1$	$(170, 176)$	$(176, 0)$
$J2/S2$	$(0, 176)$	$(6, 6)$

S1 : co-operation (black)
S2 : defection (white)



Within the regular network case, the number of defectors grew and became stable for 100% of the population

Fig. 1.6. Making the network robust against defectors' invasion by rewiring one link



of defectors increases at first and reaches roughly 60% of the population, but a rewired link may reverse this evolution in a second step. In such a case (Figure 1.6), defection decreases towards stabilisation at 11 %. Even if co-

Table 1.1. Statistical results for 500 simulations

defectors	2	3	4	6	8	17	22	36	*cycles*
purcentage	10.2	11.8	16.6	0.4	1.0	0.8	0.4	32	*16.8*

operation failed to hold in all cases of the regular network, a one link rewiring is sufficient to limit to only 1/3 the percentage of cases with a totality or a majority of defectors. Moreover, in roughly one half of the cases, defectors are limited to four or less (as in Figure 1.6). First results of simulations (Table

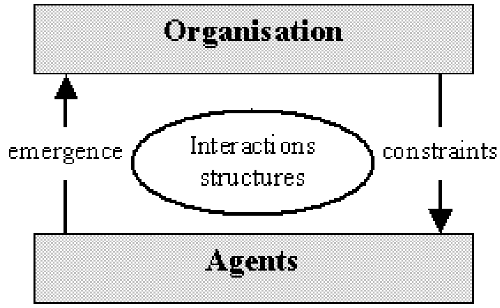
1.1) suggest that the percentage of stable co-operators becomes higher with sufficiently long range links, i.e. linked agents with a sufficiently distant local neighbourhood.

1.3.3 Basic concepts of multi-agent systems (3): emergence versus generative social science

The Santa Fe approach to complexity [3,6,16] calls *emergence* a property of a complex adaptive system that is not contained in the property of its parts. Interactions between parts of a dynamic system are the source of both complexity and emergence. In some cases, the resulting effects of interactions may seem to be random, even if they are produced by deterministic rules as in the spatial dilemma evolutionary game. An interesting part of the emergence process concerns the occurrence of some kind of order (coherent structures or patterns) as a result of the system's dynamics. This is the case with the dominance of defection or co-operation in the spatial dilemma game. In this latter case, a stable structure is the result of a selection process between pre-existing attributes of the entities (the strategies). We denote this situation as the *weak emergence* phenomenon. In other cases, the order may be a new structure which makes sense by itself and opens up a radically new global interpretation, because this does not *initially* make sense as attributes of the entities. We denote this situation as the *strong emergence* phenomenon. Strong emergence imply a morphogenetic (cognitive) process in order to include *in fine* a well identified representation of this new structure into individual and then collective consciousness.

Atlan [7] proposes a suggestive interpretation of the relationship between order and complexity, by defining complexity as "*un ordre dont on ignore le code*" (an order who code is unknown). Formally, emergence is a central property of dynamic systems based upon interacting autonomous agents. The knowledge of entities' attributes and rules is not sufficient to predict the behaviour of the whole system. Such a phenomenon results from the confrontation of the entities within a specific structure of interaction. That is, better knowledge of the generic properties of the interaction structures would make it easier to have better knowledge of the emergence process (morphogenetic dynamics). To denote a phenomenon as "emerging" does not mean that it is impossible to explain or to model the related phenomenon. For this reason Axtell (2000a) uses the word "generative" instead of "emergence" in order to avoid transcendental meaning such as in British philosophy in the 30's. Schelling's model of spatial segregation [58–60] is a precursory example of a strong emerging phenomenon, clearly based upon social interaction. Schelling's aim is to explain how segregationist residential structures (like ghettos) may occur spontaneously, even if people are not so very segregationist. The absence of a global notion of segregationist structures (like the notion of ghettos) in the agent's attributes (preferences) is a very important

Fig. 1.7. Interactions & emergent order



feature of this model. Agents have only local preferences over their neighbourhood. Moreover, people have only very weak segregationist behaviour, but the play of interactions generates global string segregationist results. In the original Schelling model, agents are localised within a 8-by-8 checkerboard (Figure 1.8). Taking the "colour" (on the checkerboard) as the criteria of discrimination, the problem of each agent is to choose a location given an individual threshold of acceptation for the proportion of other colours in their neighbourhood. That is, agents interact only locally, with their 8 direct neighbours (a so-called "Moore" Neighbourhood). There are not any global representations at all about the global residential structure.

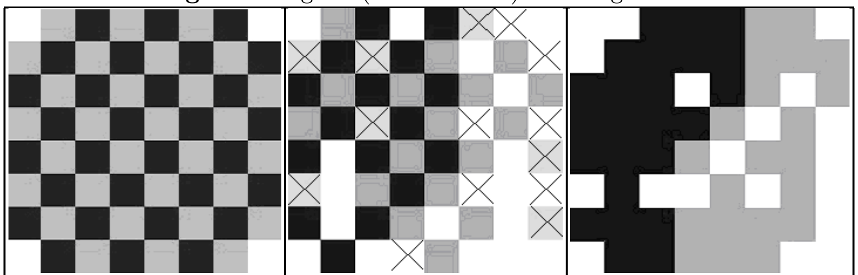
Agents have only weak segregationist local behaviour, in the following sense: each agent agrees to stay in a neighbourhood with people that are mainly of another colour, on condition that there are at least 37,5% with the same colour in the neighbourhood. More specifically, Schelling uses the following rule: an agent with one or two neighbours will try to move if there is not at least one neighbour of the same colour (with a tolerance of 50% in the neighbourhood); an agent with three to five neighbours needs at least two like him (33 %, 50% and 60% tolerance), and one with six to eight wants at least three agents of the same colour (50%, 57,1%, 62,5% tolerance).

Schelling denotes by a fully integrated structure of the population a structural pattern where there is alternately one agent of each colour in all directions; in other words, each agent (except at the edges) has four neighbours of one colour and four of the other. There is no agent in the corners. At the edges, there are two (or three) similar agents alternately among five neighbours, and two of each colour at the corners.

Under Schelling's behavioural assumption, a fully integrated structure is an equilibrium (an order) because no agent wants to move. But, from this stable configuration, a slight perturbation is sufficient to induce a chain reaction and the emergence of local segregationist patterns. Specifically, Schelling extracted twenty agents at random, and added five at random in the free spaces. By moving discontented agents, local segregationist patterns appear, like in the java applet in figure 1.8.

interactions are sufficient for the occurrence of spatial homogeneous patterns; spatial segregation is an emerging property of the system's dynam-

Fig. 1.8. Original (checkerboard) Schelling Model



a - Equilibrium with integrated population | b - 14 discontented agents (crossed) | c - convergence after 4 iterations

(source : http://www-eco.enst-bretagne.fr/∼phan/complexe/schelling.html)

ics, while spatial segregation is not an attribute of the individual agents. Sometimes, integrated (non-homogeneous) patterns may survive. Integrated structures are easily perturbed by random perturbations, while homogeneous structures are more stable (frozen zones).

In Figure 1.8b, the discontented agents are shown by crosses. These agents move at random towards a new location in agreement with their preferences. This move generates new discontented agents by a chain reaction until a new equilibrium is reached. This may be a state of perfect segregation, with clearly delimited ghettos, like in Figure 1.8c, or locally integrated patterns may survive in some niches within homogeneous patterns of populations.

In the Schelling model, ghetto formation is the *non-intentional* result of the composition of individual behaviour. The local intention (preference) of the agents is not to be too isolated. Agents do not want to create a new organisation of space. Such a structure is said to be "emerging" because it is not an attribute of the chosen space of the individual agents before this kind of order emerges. In other words, agents do not choose between a segregated spatial arrangement or an integrated one. They only randomly move whenever they are discontent. A segregated spatial pattern is not the consequence of the behaviour of a particular agent, but all the agents contribute actively or passively to the emergence process, through social interactions. In the Moduleco multi-agent platform, agents are really mobile over the locations. The main results of the Schelling model are robust over different algorithms for the agents' moves and different sizes of the network.

The creative principle of emergence is a central property of complex adaptive systems . But the temporal effects of interactions upon structures do not appear necessarily as homogeneous. One may observe long periods of stability (punctuated equilibrium) separated by periods of crisis.

In a "linear" world, the proportionality principle applies by associating small effects to small perturbations, while major perturbations are necessary to generate significant break down. In an interactive world, dynamics are mainly non-linear. The principle of proportionality is no longer valid and dynamics are generally non-linear. Similar magnitude changes in some parameters' or agents' attributes may produce very different magnitudes in the system's reaction like, for instance, when chain reactions and/or events like phase transition occur (see Gordon, Galam in this book and the effect of price change upon customers' behaviour in the next section).

1.4 Individual and collective learning and dynamics in a discrete choice model

Given the classical subdivision of cognitive economics into an *eductive* (individually centred) and an *evolutionist*' perspective, one interest of ACE is to allow us to integrate both dimensions in the same framework. On the one hand, with CL such as Moduleco, it is easy to model population dynamics with adaptive agents. On the other hand, the conceptual and formal integration of both dimensions within a significant and coherent framework is a real challenge.Taking the market as a complex and cognitive informational and interactive system, this section presents two models of a monopoly market with discrete choice [5]. The first one focuses upon individual learning at the monopolist level, in an interactive decision theoretical approach, with Bayesian features. The second one focuses upon collective learning at the market level, where individual demands are related though social influence within a communication network. In each dimension taken separately, dynamics considerations are far from being trivial, and CL appears to be a useful tool to investigate numerous variants of given problem by simulations , where an exact solution exists only in the very simple case.

1.4.1 Individual learning and the exploration-exploitation dilemma

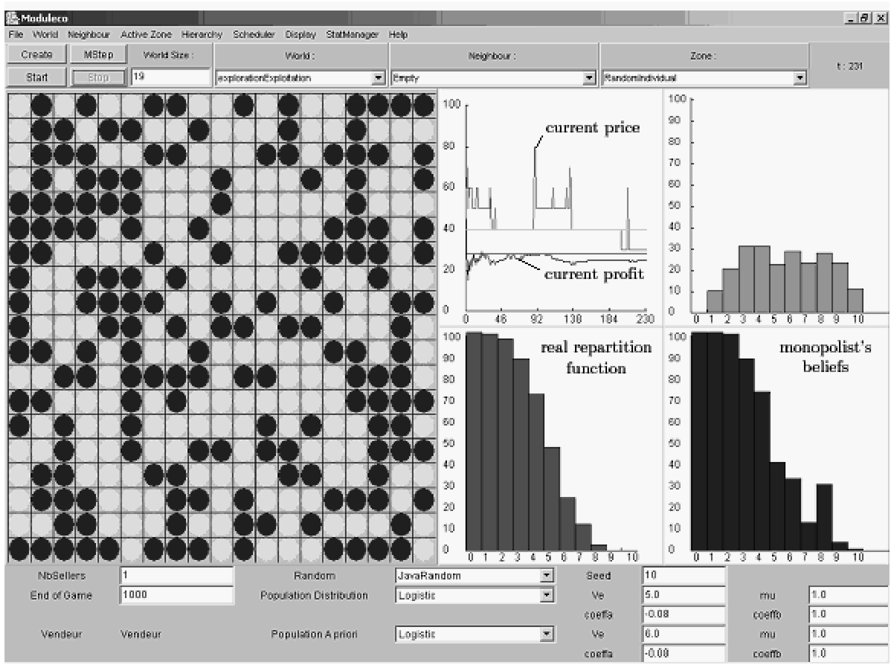
In order to illustrate individual learning in a market simulated on Moduleco, Figure 1.9 presents the graphic interface of a model by Leloup [40–42] and [43] of dynamic pricing based on optimal Bayesian learning by exploration-exploitation arbitration, using the Gittins Index [24,13].

In this model, we have a monopolist faced with heterogeneous customers whose individual reservation prices are non observable. In the simplest case, the distribution of such reservation prices is initially known, except for a given parameter. In a more uncertain case, the distribution itself is unknown, but the monopolist has some belief about these distributions. Potential customers make binary choices (to buy or not) and the monopolist has some a priori beliefs upon the statistical distribution of such reservation prices. More specifically, the monopolist sequentially matches a single potential customer taken at random at each iteration. At each iteration, the monopolist can charge a price that belongs to a discrete, ordered, finite set $\{p_1, ..p_i, ..p_k\}$, where p_1 is the minimal price and p_k the maximal one. These prices do not allow the agents to bargain : either the agent buys at this price, or does not buy. That is, assuming a null cost without loss of generality, at each period t, the monopolist has a profit of $\pi_t = p_t$ if the agent buy, or zero otherwise. Sequential profit flows are subject to a geometric discounting along an infinite time horizon. In this model, idiosyncratic preferences of agents are given at the beginning of the process. In order to be coherent with the following model, let us assume a logistic distribution for the willingness to pay. Without social influence between agents, the resulting distribution is stationary. Let us remark that, in such a framework, the monopolist's problem is not to really to learn the initially unknown true parameter of the distribution (the case of prior belief about distribution corresponds to the true - here logistic - distribution). The monopolist's dynamic problem is to maximise his discounted profit by taking a locally optimal decision given the available information, according to a Bayesian decision approach. As in the pioneering model by Rothschild [57], in this dynamic approach, incomplete learning may occur. This means that for an infinite period of time (and a fortiori in a finite period of time), a seller following an optimal Bayesian learning process by exploration-exploitation arbitration at each point in time may obtain a sub-optimal result. Moreover, self-reinforcing machine learning processes or other adaptive learning procedures which are generally non-optimal at each point in time may produce better results in some cases (more actualised cumulated profit). This kind of result may arise because with actualisation, strong profit in the starting process are more valorised than well fitted asymptotic results. For instance, efficient maximum likelihood estimation ensures adequate learning of the true parameters, but requires costly information not available at the beginning of the market process.

Computational complexity raised by dynamic programming in this case is well known, even in the case of non sophisticated behaviors. In order to overcome this cognitive and computational problem [40] introduces a non-parametric discrete approximation technique called “beta-logistic” . This approach is based on the following observation: the sequence of profits that are associated with the various prices offered by the monopolist are Bernoulli samples. In this context, the unique formulation of the monopolist's prior beliefs which permits a joint analysis of his learning process is the family of beta distributions. As a result, non-parametric estimation of the distribution of reservation prices over the potential customers may differ significantly from a logistic curve, even in the case where the true distribution is logistic. That is the case in the simulation under review, where prior beliefs of the monopolist follow a logistic distribution, which is projected on the beta distribution family. Although such a method may seem strange in the case where the prior and the true distributions are the same (except for some unknown parameters), it appears to be powerful in the maybe more realistic case, where the real distribution is non-parametric or different from the prior one.

In the simulation in Figure 1.9, the size of the population is 192 361 agents. The true logistic distribution, with parameters $V_e = 5$ and $mu = 1$, has a cumulative representation in the south-west quarter. The dispersion parameter, mu, is assumed to be known. $V_e = 5$ means that 50% of the population of agents buy for $p = V_e.10 = 50$. The prior and (non parametric) updated distribution is represented in the south-east quarter. The unknown parameter is estimated by the way of prior belief as $V_e(0) = 6$, which means that the monopolist is optimistic. That is, he believe that 50% of the population of agents buy for $p = V_e(0) * 10 = 60$. At the beginning of the period, an omniscient monopolist, who would know the true distribution, would charge the optimal price at $p = 40$, and the related profit would be 30. These two references are drawn by lines on the north-west quarter. The uneven curve above the line at $p = 40$ is the effective trial and error optimistic price. After roughly 40 iterations, the monopolist finds the " good" price and maintains this price over 50 periods (exploitation). A new temporary re-exploration of higher prices arises. Such exploration can be interpreted in the following way : the monopolist is not sure about the profitability of these (higher) prices already charged in the past and does it again. Because these higher prices decrease the cumulated profit, the price returns to $p = 40$, for a new transitory period of exploitation etc. In this model, on the one hand, only the seller

Fig. 1.9. Optimal learning by experimentation



has a significant cognitive activity, and on the other hand, one can explore the effect of communications structures between agents. Leloup [40–42] extends this framework to a dynamic pricing model in which the buying agents are able to communicate their purchase experience to other buying agents. Agents are assumed to have an ad hoc revision policy for their reservation price which consists in rejecting all prices that are strictly higher than a price that has been charged (in the past) by the selling agent to a member of their neighborhood, even if these prices are lower than or equal to their initial reservation price. In the case of a Moore neighbourhood, because the diffusion of the information between customers, the probabilities of purchase associated with high prices rapidly decrease if the monopolist explores lower prices to inquire about their profitability. Moreover, when the monopolist has pessimistic prior beliefs, the price dynamics converges towards a price that is often less than the initial optimal price. Finally, in this setting, the cumulative distribution function of willingness to pay is no longer stationary. The resulting complexity of such a problem renders the analytical study of price dynamics hard to carry out, and ACE allows us to get insights into the characteristics of such a market.

1.4.2 Collective learning and complex dynamics in a discrete choice model with networked externality

Phan, Pajot, Nadal, [56]. explore the effects of the introduction of localised externalities through interaction structures upon the local and global properties of the simplest market model: the discrete choice model (Anderson et al., 1992) with a single homogeneous product and a single seller (the monopoly case). The general characteristics of this model are studied by [51] see Phan et al., this book for a synthesis and relationship with other models of social influence as well as the statistical mechanism). We focus here on the dynamics of the demand based upon both individual idiosyncratic preference and networked social influence, with exogenous prices. The ACE approach allows us to investigate both the price-dependent equilibrium path and out of equilibrium market dynamics and to underline in what way the knowledge of the generic properties of complex adaptive system dynamics can enhance our perception of such market dynamics.

In this model, the agent has to choose between buying ($\omega_i = 1$) or not buying ($\omega_i = 0$) one unit of a given goods. Agents are assumed to have a linear willingness to pay, and maximise a surplus function $V_i(\omega_i)$. That is, their individual choice makes $V_i(\omega_i)$ positive if the agent buys and null otherwise.

$$\max_{\omega_i \in \{0,1\}} V_i = \max_{\omega_i \in \{0,1\}} \omega_i (h_i + J_\vartheta \sum_{k \in \vartheta_i} \omega_k - p) \tag{1.1}$$

Specification (1) embodies both a “private” and a “social” component, which correspond to the idiosyncratic and the interactive heterogeneity respectively. The private component h_i is strictly deterministic at the agent level (see Phan et al., Chap. **??** this book for a discussion of this assumption). To be more significant, let us decompose this first component between a common sub-component h, and an idiosyncratic sub-component θ_i, such as : $h_i = h + \theta_i$. Agents are randomly distributed on the network (fixed random field) according to a parametric cumulative distribution $F(z)$ with mean = 0 (more specifically, θ_i are logistically distributed with variance $\sigma^2 = \pi^2/(3\beta^2)$):

$$\lim_{N\to\infty} \sum_N \theta_i = 0 \Rightarrow \lim_{N\to\infty} \frac{1}{N} \sum_N h_i = h \tag{1.2}$$

The social (or interactive) component embodies additive effects of the choices of the others upon the agent's choice. Specification (1) does not have an unequivocal semantics. That is, numerous cases, including latent submodels, can lead to such linear social interdependence. Formally, assuming a regular network and homogeneous interactions in each neighbourhood, we have symmetric $J_{ik} = J_\vartheta = J/n_\vartheta$ for all influence parameters, where n_ϑ is the number of neighbours around agent i and J a positive parameter. For a given neighbour k taken in the neighbourhood ($k \in \vartheta$), the social influence is J_ϑ if the neighbour is a customer ($k = 1$), and zero otherwise. That is, social influence depends on the proportion of customers in the neighbourhood. For physicists, this model is formally equivalent to a "Random Field Ising Model" (RFIM - see Galam, Chap **??**, and Phan *et al.* Chap. **??**, this book).

In this class of models, the individual threshold of adoption implicitly embodies the number of people each agent considers sufficient to modify his behaviour, as underlined in the field of social science by Schelling [60] and Granovetter [25], among others. In this case, the adoption of a single agent in the population may lead by chain reaction to significant change in the whole population. Taking an example of an incremental change in price, a chain reaction may link one or several other "direct adopters" with " indirect adopters" . Adoption by the former is only motivated by a change in the so-called "external field" : $H_i = h_i - p$, for a given value of the social influence (the "local field"). Formally, those with surplus function such as: $h_i + S(p-1) - p > 0$, where $S(p-1)$ denotes the value of the local field before change in price. The latter may be motivated by changes both in the external field and the local field (social influence). However, any "indirect adopters" would change their behaviour without taking into account the social influence effect; that is, they have : $h_i + S(p-1) - p < 0$, but chain reaction conduct the local field towards a value such as the surplus became positive. In the following, the word " avalanche" refers, at the global level, to the cumulative effect of such a chain reaction until reach the next equilibrium.

When individual choice depends upon social influence , two kind of dynamics characterise such avalanches. On the one hand, if all agents take into account only the global mean choice of the others, the situation is formally equivalent to the so-called "mean field" approximation of the physicist. That is, for sufficiently large populations, "global" interaction is equivalent in specification (1) to complete interconnection ($n_\vartheta = N-1$), because the normalisation assumption $J_\vartheta = J/n_\vartheta$ leads each individual to be influenced to the same magnitude by the mean choice of the others (the "world" neighbour in Moduleco). In this case, because social influence is "as if" , the neighbourhood of each agent would be composed of all the other agents. Both avalanches and aggregate demand are independent of the topology of the social network. On the other hand, local interdependence gives rise to localised avalanches on the network, following the structure of the network. Characteristic related consequences are the emergence of clusters with possible locally frozen zones (Galam, this book).

Starting from an initial situation where any agent has adopted the product ($\omega_i = 0$ for all i), if the idiosyncratic component of willingness to pay were uniform, ($h_i = h$, for all i), each agent' choice depends on the sign of the external field: $H = h - p$. In such a case, one could have a so called "first order transition" , if all the population abruptly adopted it, when p decreases below h. Let $p_h = h$ denote this take-in threshold. It is significant to observe than the inverse phenomenon do not have the same threshold, because when all agents are previously adopters, the surplus function depends both on the external field and on the local field. The latter is equal to J, because all agents are adopters. As a consequence, the take-off threshold will be $p_j = h + J$: if the price decrease under p_j, all the agents are no longer customers for this good, and the whole population abruptly leaves the market. As a consequence, in such an extreme case, after adoption, there exists a price interval $[p_h, p_j[$ within no change occurs in the market demand.

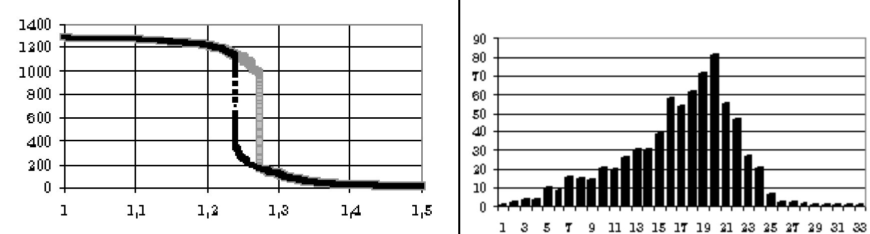
In the presence of *quenched disorder* (non uniform h_i), hysteresis loops may occur. The number of customers evolves by a series of cluster flips, or avalanches. If the disorder is strong enough (the variance σ^2 of h_i is large compared to the strength of the coupling J), there will be only small avalanches (each agent following his own h_i). If σ^2 is very small, then there is a unique "infinite" avalanche, as in the uniform case previously described. There is an intermediate regime where a distribution of avalanches of all sizes can be observed.

From the theoretical point of view, it is possible to identify, a special price value p_n, which corresponds to the unbiased situation. In this case, on average, the willingness to pay is neutral: there are as many agents likely to buy than not to buy. Formally, if only 50% of the agents are customers, the average willingness to pay is $h + J/2$, and $p_n = h + J/2$. Let us remark than pn is exactly the middle of the price interval $[p_h, p_j[$. Starting from this unbiased situation, where $p = p_n$ For $p < p_n$, there is a net bias in favour of "buy" decisions ($h + J/2 - p > 0$), whereas for $p \geq p_n$ there is a net bias not in favour of "buy" decisions. A spontaneous symmetry breaking occurs when an avalanche leads from a situation where $p < p_n$ with less than 50% of customers towards a situation whith more than 50% of customers, even with this lower price.

To experiment such a phenomenon, it is useful to take a simple example from a simulation. Let us take a logistic distribution with mean=0 for the cumulative distribution $F(z)$ (see Phan et al., this book for a discussion). For a given variation in price, it is possible to observe the resulting variation in demand. The most spectacular result is when nearly all agents update their choices simultaneously ("world" - synchronous - activation regime) , in the case of global interactions (complete connectivity). in Figure 1.10a, curves plot each step in the simulation for the whole demand system, including the set of equilibrium positions for a given price. The black (grey) curve plots the "upstream" (downstream) trajectory, when prices decrease (increase) incremented in steps of 10^4, within the interval $[0.9, 1.6]$. We observe a hysteresis phenomenon with phase transitions around the theoretical point of symmetry, $p_n = 1,25$. In both cases, strong avalanches occur in a so-called "first order phase transition".

Along the upstream trajectory (with decreasing prices – black curve), a succession of growing induced adoption arises for $p = 1.2408 < p_n$, driving the system from an adoption rate of 30% towards an adoption rate of roughly 87%. Figure 1.10b shows the chronology and sizes of induced effects in this dramatic avalanche.

Fig. 1.10. Straight phase transition under "world" (synchronous) activation regime



a - Hysteresis in the trade-off between prices and customers: upstream (black) and downstream (grey) trajectories

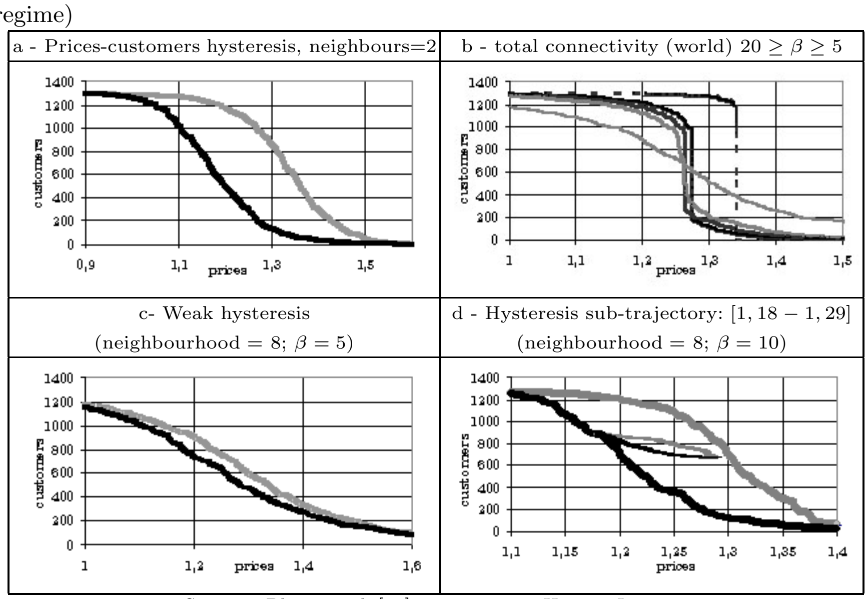
b - Chronology and sizes of induced effects in an avalanche at the phase transition for $P = 1.2408$ ($P_n = 1.25$)

(source: Phan *et al.* [56]; parameters: $H = 1$, $J = 0.5$, Logistic with $\beta = 10$).

Along the downstream trajectory (with increasing prices –grey curve) the externality effect induces a strong resistance of the demand system against a decrease in the number of customers. The phase transition thresholdis here around $p = 1.2744 > p_n$. At this threshold, the equilibrium adoption rate decreases dramatically from 73% to 12,7%.

The scope of avalanches within the hysteresis loop increases with connectivity. Figure 1.11a exhibits a soft hysteresis loop (called second order phase transition) with the same parameters, but within a regular (periodic) network in dimension one, for two neighbours. As suggested by the previous example of no idiosyncratic willingness to pay ($h_i = h$ for all i), the steepness of the phase transition increases when the variance of the logistic distribution $\sigma^2 = \pi^2/(3.\beta^2)$ of the θ_i decreases (when β increases). The closer the preferences of the agents, the greater is the size of avalanches at the phase transition. Figure 1.11b shows a set of upstream trajectories for different values of β taken between 20 and 5. For $\beta < 5$ here is no longer any hysteresis at all. Figure 1.11c shows a narrow hysteresis for a regular (periodic) network in dimension one, with eight neighbours, while Figure 1.11d exhibits a larger one. Finally, following results by Sethna [62], inner sub-trajectory hysteresis can be observed in the case of this Random Field Ising Model (Figure 1.11d). Here, starting from a point on the upstream trajectory, an increase in price induces a less than proportional decrease in number of customers (grey plot). The return to the exact point of departure in the case of decreasing prices again (black curve) is an interesting property of the Sethna's inner hysteresis . From the economists' point of view, such a property may be used by the seller in an exploration-exploitation process of learning around a given trajectory.

Fig. 1.11. The trade-off between prices and customers (synchronous activation regime)



Source : Phan *et al.* [56]; parameters: $H = 1$, $J = 0.5$

To conclude, in the case of regular networks, a discrete choice market with externality provides, numerous complex dynamics on the demand side. As a result, the seller's problem is generally non trivial, even in the case of risk, where the seller knows all the parameters of the program (1) and the initial distribution of the idiosyncratic parameters (Phan *et al.*, this book). In particular, an interesting challenge for cognitive economics is to try to merge the exploration-exploitation Bayesian revision process in a sequential discrete choice model without externality , reviewed in (31) and the externality case (32), which raises the question of the non-stationary environment of both the upstream and downstream trajectory.

1.5 Conclusion

This chapter is an attempt to provide an introduction and easy understanding of typical complex phenomena that may arise in interactive context modelling by way of ACE. Moreover, Computational Laboratories (CL) provides a useful framework to friendly model, understand and investigate the dynamics of complex adaptive systems. Both ACE and CL are therefore, very useful for modelling markets viewed as cognitive and complex social interactive systems, in the way of cognitive economics.

The last section presents two models in the simplest monopoly market case: discrete choice with a homogeneous product. The former focuses upon individual learning at the monopolist level, in an interactive decision theoretical approach. The latter focuses upon collective learning at the market level, where individual demand are related though social influence within a communication network. In both cases, addressing separately one dimension of cognitive economics, the resulting dynamics are far from being trivial, and CL appears to be a useful tool for investigating such problem by simulations , where an exact solution may exist only in the simplest case.

The integration of both the collective and individual dimension in the same framework is a real challenge for cognitive economics. Actually, even if it is easy to model population dynamics with adaptive agents in an ACE framework, the conceptual and formal integration of the two dimensions within a significant and coherent analytical framework need more development. If we want to keep a link between analytical and ACE modelling, the connection between the two dimensions need such integration in simple cases, such as the reference and departure points. Without such a reference, ACE will be widely disconnected from a more standard approach. Such a disconnection is a possible issue for modelling economic problems, where ACE would be a complete substitute for an analytical approach. The strategy suggested here is *to keep the connection between these two approach and to use ACE as a complement of the analytical one*, in particular to investigate complex dynamics linked with both social interactions and belief revisions. Unfortunately, cognitive economy, which provides powerful models separately in an eductive and an *evolutionist*' perspective, fail at this time to provide an integrated analytic framework of reference. Let us note the advances by Orléan (this book), in taking into account the collective dimension of belief, through his discussion on the nature of social representations. However, the integration of the two dimensions seems to be the major challenge for the coming years.

Finally, numerous interesting cognitive economics questions to model by means of the ACE framework are not reviewed here. In this book we can cite among others, the emergence and dynamics of networks (Bloch, Curien et al., Galam et al., Weisbuch et al., Zimmerman…), viability and control (Aubin), evolutionary games models (Baron et al., Laslier). In the issues not addressed here, co-evolutionary dynamics for populations of agents heterogeneous with respect to their cognitive capacities [12] will also be stimulating challenge for both ACE and cognitive economics in the years to comes.

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http://www.econ.iastate.edu/tesfatsi/ace.htm

Multi-Agent Platforms:

Ascape	http://brook.edu/es/dynamics/models/ascape
Cormas	http://cormas.cirad.fr/fr/outil/outil.htm
LSD	http://www.business.auc.dk/∼mv/research/topic_Lsd.html
MadKit	http://www.madkit.org/
Moduleco,	http://www-eco.enst-bretagne.fr/∼phan/moduleco/
Swarm	http://www.swarm.org/

Acknowledgement - I acknowledge FT R&D and GET (Group of Telecommunications Engineering Schools) for financial support and Antoine Beugnard for the architectural design of Moduleco. We thank Nigel Gilbert and the University of Guilford; Marco Valente, Luigi Marengo, Corrado Pasquali and the University of Trento, for their material or intellectual contribution to the early development of Moduleco; Marc Barthelemy, Paul Bourgine, Jean Louis Dessalles, Jacques Ferber, Mirta Gordon, Benoit Leloup, Jean Pierre Muller, Jean Pierre Nadal, Séphane Pajot, Cyrille Piatecky, Michel Plu, Thomas Vallée for valuable discussions and intellectual support; the participants of the cognitive economics seminar, specially Richard Baron, Alan Kirman, Bernard Ruffieux, Gérard Weisbuch for their comments on a preliminary version of this Chapter; and finally all the programming contributors of Moduleco for their help.